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Bulletin of American Association of Jesuit Scientists

EASTERN STATES DIVISION

VOL. XXIII MARCH, 1946 No. 3

OBITUARY

FATHER FRANCIS J. DORE

1876 – 1944

R. I. P.

Father Francis J. Dore, S.J., was born in Boston March 7, 1876 and after the completion of his course at Boston College High School entered the Society at Frederick in August 1893. He left Frederick in January 1894 and returned to study at Boston College and Harvard Medical School. He received his M.D. in 1901, and made his internship at the Carney Hospital in South Boston. After a few years in medical practice he reentered the Novitiate at Roehampton on April 25, 1907 and returned to Woodstock for philosophy. He taught at our college in Brooklyn for a year, made his theological studies at Woodstock and was ordained in 1916. He taught at Regis High School in New York in 1917 and 1918 and at Fordham Medical School during the academic year 1918-1919. In 1920 he finished tertianship at Poughkeepsie and was appointed minister at Fordham University. The next year he was Regent of the School of Social Work and also of the Pharmacy School at Fordham. For three years, from 1922-1925, he was Socius to the Master of Novices at St. Andrew-on-Hudson.

As a native of Boston he was assigned to the New England Province in 1925 and returned to Boston College where for eighteen years he was Head of the Biology Department and Director of Pre-Medical studies. For many years he also taught Biology at Weston. During the same eighteen years he was the faithful chaplain of the Academy of the Sacred Heart in Newton.

In the summer of 1935 he was chosen chaplain of the Pan-American Medical Association Congress Cruise to Brazil and the West Indies. On Feb. 5, 1936 he was elected Honorary Member of the Massachusetts Medical Society.

In 1943 when the Army took over St. Mary's Hall he moved with other members of the Boston College Community to a house on Beacon Street. As he was crossing Beacon Street one night in November he was struck by an automobile and seriously injured. He suffered patiently in St. Elizabeth's Hospital for more than three months and received many blood transfusions from Ours and from our students and Alumni. On February 28, 1944 God mercifully released him from his pain and full of merits he passed to his reward. His funeral at Weston was attended by a large number of the clergy of the Archdiocese and many doctors, his former students, who were most devoted to him. His younger brother Father Leo A. Dore, S.J., celebrated the funeral Mass.

SCIENCE AND PHILOSOPHY

SOME NOTES ON SPACE

Rev. Joseph P. Kelly, S.J.

The term "Space", like Time, is quite commonly used in daily life. The astronomer tells us that the heavenly bodies are moving through space. The realtor advertises "space to-let" in office-buildings and warehouses. In various parts of the city the motorist reads the inviting signs: "Parking Space". Though many might find difficulty in trying to form a strict definition of the term, all would agree that they understand, at least in a general sense, the meaning of the word, Space. It seems to connote the idea of extension, in the sense of area or volume. A business man who considers "space to let" in a warehouse, might justly figure this in terms of a number (or volumes) of bales of wool. A parking lot would be thought of in terms of square feet. And a further connotation of the word would convey the notion of occupation of this extension. So that an extension capable of being occupied would seem to be included in the ordinary comprehension of the term, Space.

The Scholastic Philosophers discussed the notion of Space at great length. (1). Their analysis led to the conclusion that Space is based upon or at least is intimately connected with the extension of real, physical bodies. Physical bodies are extended. They are not concentrated in mathematical points. One part of the body is outside of the next part. Therefore they have dimensions, three dimensions, length, width and depth or height. There is a dimensional interval enclosed within their physical limits or surfaces. If we consider a body existing somewhere and imagine it removed from that place, we have left an interval, an emptiness—we prescind from the atmosphere—and this interval may be filled by another body of the same dimensions or by a number of bodies successively, of similar dimensions. This void is nothing in itself; it is not a real entity like a house or tree. We may represent it to ourselves as a receptacle for these bodies. It "receives" these bodies, according to our way of thinking of it, somewhat as a glass receives water that is poured into it. Of course, the glass is a real being but

(1) These notions of Space are based on Suarez, "Disputationes Metaphysicae." Disp. LI. Sect. I; no, 11sq. Sect. II. no 24. c.f. also "Jesuit Science Bulletin" May, 1936. "Suarez and Einstein on Space," by Rev. J. W. Ring, S.J.

the emptiness is nothing. This vacuity, capable of receiving bodies is the notion of Space. Space is a vacuity, an empty interval, with a capacity for being filled by real, dimensional bodies. We look about and see other bodies and we are able to repeat the process. If we look about us into the heavens, say from the earth to the sun, we recognize that the empty interval could easily be filled by other bodies like the earth. This emptiness can be considered as capable of being occupied by many thousands of real bodies. It is a non-being but considered as a huge receptacle for real bodies. This non-being, represented as if it were something, constitutes the notion of Space. Space then, is a vacuity, represented as a capacity for physical bodies. It is important to note that in this concept we should place emphasis on the capacity of bodies to fill the vacuity and not on the vacuity itself. The notion is derived from the extended beings. It is their capability to fill the void that leads to the fillable void, which is Space. Hence, the concept of Space is constituted of a vacuity and a capacity to be occupied by real, extended bodies. In Scholastic terms, this concept is a figment of the mind with a foundation in reality. The vacuity is a non-entity but is represented as a something by reference to the real bodies that may or actually do occupy this void. The concept is partly real and partly ideal. In starting from the real beings, some Scholastics prefer to take distance as the foundation-notion. In reality this becomes a question of terminology bceause the distance may be equally conceived as an interval, devoid of reality but capable of being filled or occupied by dimensional bodies.

There are other opinions among Philosophers that are not in agreement with the Scholastic concepts. Kant and the Kantians hold that Space is a purely mental concept. Kant held that we think of bodies as spatial by a necessity of the mind. The concept is innate; it is not derived from the exterior bodies, but, by a law of the mind we, of necessity, think of them in a spatial manner. In this opinion, we cannot attribute anything of the notion of Space to objects. Space is a "mental category" whose function is to represent physical beings in a spatial manner. To say that bodies exist in Space or move in Space is a merely mental expression. We cannot agree with Kant's doctrine because it seems to contradict experience and gives no account as to why we should have this notion of Space as part of our mental equipment. Clarke and Newton went to the opposite extreme attributing to Space an extramental reality of its own. They considered it a real being, really existing like other realities such as the earth. While this ultra-real notion was quite suitable as a background-concept for Newtonian Mechanics, we have no proof of its reality and to-day, most of the scientists reject the reality of Space. In the philosophy of Descartes, Space was identified with extension. That was a logical position for Descartes because extension was a fundamental notion in his outlook on the physical world. Some others despaired of solving the problem of Space and placed it among the enigmas of the universe.

Various Kinds of Space

If we consider Space merely as an interval capable of being filled or occupied by physical bodies, we have what the Schoolmen called *Possible Space*. For example, the Space or interval between the earth and the sun, considered merely as a capacity or receptacle for receiving heavenly bodies, would be Possible Space. *Real Space* would be any part of Possible space which is actually occupied by a real body. For example, the part of space occupied by the earth or by the Empire State Building in New York. Such space would have actual dimensions which are the real dimensions of the body occupying that space. If we consider Pure (*) and Possible space together, we call it Absolute Space. Some call it Imaginary Space. This Space extends without limits in all directions. It is that void that we see when we look into the heavens, the Space that we conceive as existing before the creation of the world. Or to put it in another way, if we were to imagine all the heavenly bodies annihilated, excepting the earth, and look out into that broad expanse extending above us and to all sides, indefinitely in all directions, this would be our picture of Absolute Space. This should not be confused with Geometrical or Mathematical Space. Geometry is the Science of Space, it is asserted. That space is abstract extension considered in itself. Propositions in Geometry have nothing to do with, in fact prescind from the question of the occupation of space by bodies. In the formation of the concept of Geometrical Space, we start with extended bodies. We extract from them the notion of extension, in itself. This abstract extension is the subject matter of Geometry; its propositions deal with this extension or space. It does not enter into the question of occupation or non-occupation of space by bodies.

Notions of Space are based on our ordinary experience; they are ideas derived through our sense perceptions under ordinary conditions of perception. They are Euclidian in nature. It is the only type of Space to be found in this way because we can have no experience of other bodies than those which exist in the world about us. Bodies are three dimensional and our spatial concepts will be three dimensional, supposing their foundation in sense perception. Time is said to be a fourth dimension but it is not of the same nature as a spatial dimension. The question may be asked whether there are spaces of more than three dimensions. If there are, we cannot have direct experience of them through the senses. In a later paper, we will have more to say about multi-dimensional space. (2). When we discuss one or two-dimensional space, as in a line or a surface, we do so by an act of the intellect, prescinding from the other dimensions, for a line or a surface

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(*) Pure Space is the same as Geometrical or Mathematical Space. It is extension considered in itself.

(2) Jesuit Science Bulletin. Oct. 1937. "Some Philosophical Aspects of Multi-dimensional Space." Rev. R. B. MacDonnell, S.J.

as such do not actually exist. In the physical order only three dimensional bodies exist.

Closely connected with the notion of Space, is the concept of Place. It is also called Ubication or Location. It answers the question: Where is it? From ordinary observation, seeing some bodies move upwards, and others downward toward the centre of the earth, the Medieval Schoolmen concluded that every body had its natural Place to which it moved by a sort of impulse of nature. This Place or Location was necessary for the concept of local motion and distance. Local motion in a body is the movement from one place to another. Place is the term, initial and final, of motion. Likewise, distance is measured from place to place, e.g., Boston to New York. In the method of modern science, a moving body is really considered as a point, (the matter of a body concentrated at the centre of gravity), moving from one point, (initial place), to another point, (terminal place). This is a convenient fiction for the sake of accuracy. The Philosophers looked at the problem from a qualitative point of view, and divided Place into *Internal* and *External Place*. The Internal Place is that portion of space occupied by a body. It is a definite part of space, having the actual dimensions of the body filling it. In this sense, Place is a quality of a body, a localizing accident, which fixes a body in a definite part of space. In a body at rest, it is considered as something permanent.

The Scholastics recognized that these notions were not sufficient, since they were too indeterminate for the definite location of bodies. They might be anywhere according to this meaning of Internal Place. So, they added External Place to the explanation, a reference to the surrounding objects. Place, then, is that portion of space occupied by a body, but considered in reference to surrounding bodies. It is a relative quality of a body, connoting a relationship to other objects. Hence they said, if only one body existed, e.g., the earth, it would have no place, according to their definition. There would be no surrounding bodies. In modern parlance, if one were to ask whether a single body could have a place, it would be considered a "meaningless question." The External Place of a body was defined as "the immobile surface of the surrounding matter." The place of a ship is the surface of the water in contact with the surface of the ship. The place of a rock imbedded in a mountain would be the immediate surface of the earth surrounding the rock. Fair enough for bodies at rest but not very clear for moving beings. An aeroplane flying through the air is changing its position constantly, and equally so, a ship moving at sea. The definition states that the surface should be immobile. The philosophers saw that something was needed to give value to the definition. The "immobile" must be relative. Hence, moving bodies were referred to the surrounding surface and further to the centre of the

earth or to the poles which were considered as fixed points. Others referred to the seashore as the fixed point.

Thus, Space, Place, Location and Distance are closely bound up with the notions of Space. Modern Science deals with these ideas but mostly from the point of view of measurement, while the Scholastics took the qualitative viewpoint. This outlook does not offer all the precisions and accuracy of measurement that we find in science. However, it does furnish concepts which may well be the foundation measurable determinations.

EDITOR'S NOTE.—The two articles on TIME, c.f. Jesuit Science Bulletin, Vol. XXII, p. 36, and p. 97, and the present article on SPACE treat these topics from the point of view of Scholastic Philosophy. There will follow a discussion of these questions in their relation to Space and Time in Modern Physics, by Rev. Thomas H. Quigley, S.J., Prof. of Physics at Weston College, and Rev. Joseph P. Kelly, S.J., Prof. of Cosmology at Weston College.

ASTRONOMY

THE NORTH POLAR SEQUENCE

Rev. F. J. Heyden, S.J.

The early survey catalogues of stars aimed chiefly at deriving exact positions of the stars. Magnitudes played the subservient role of aides to identifying the stars corresponding to the given positions. (1) Hence accurate magnitudes were only accidental, and there existed no definitely established scale or zero point upon which all estimates could be based.

From earliest times the naked eye had been the principal photometric instrument and while no strict attention was paid to an accurate scale for differences in brightness between stars, still the eye obeyed a law from which a scale could be derived. This law known as Fechner's Law, is empirical and states that the intensity of visual sensation varies as the logarithm of the strength of the light stimulus. The normal eye will discern a difference in two light sources amounting to only 1%.

In 1850, the same year in which the image of a star was photographed for the first time at Harvard, Pogson proposed the first standard scale for stellar magnitudes,

$$m = - 2.5 \log I + C$$

where m signifies the apparent magnitude, I the intensity, C a constant determining the zero point, and 2.5 was derived from the fact that in the oldest magnitude scale the sixth magnitude stars were roughly 100 times fainter than a first mangnitude star. Hence,

$$\frac{I_1}{I_6} = 2.512^5 = 100$$

The zero point of Pogson's proposed scale was to be determined by the 6th magnitude stars of the Bonn Durchmusterung.

Not until about 1880 was there any effort made to establish this scale once and for all by selecting a certain group of stars as reference

standards. At this time it was proposed by the American Association of Sciences (XXXIII, 8) that a group of stars in the north polar region be chosen for this purpose. Accordingly 21 stars, many of which are still members of the North Polar Sequence, were set apart and magnitudes estimates to the first decimal place. (A list of these stars is given in a paper by E. C. Pickering entitled "An Investigation of Stellar Photography," 1886).

With the advent of faster photographic plates (the first photograph had been taken on a daguerrotype) Professor Pickering made several trial exposures on the polar stars. The photographic magnitudes determined from the plates showed a degree of consistency between successive observations that exceeded any degree of accuracy previously attained by other methods of photometry. Single measures of photographs taken on different nights differed on the average by less than a tenth of a magnitude.

Accordingly a program was undertaken to determine the photographic magnitudes of the polar stars proposed by the American Association. As might be expected, a difference between photographic and visual magnitudes was found, but at this stage neither the visual or photographic scales had been well enough determined to show a reliable color index. In fact the stars of the sequence were not all of the same color.

The effect of color upon the scale of magnitudes was appreciated from the very beginning. Pogson's scale had been accepted for visual magnitudes, and the first experiments with photographic methods indicated that the same scale could be used provided proper allowances were made for the color of the stars. Unfortunately, in the early beginning spectra and colors of stars were not well known and space reddening was not even suspected.

Around 1888 the work on the North Polar Sequence really began. The first accurate results appeared some time later in H.C.* 125. This first standard consisted of only ten stars. The North Polar Sequence was not considered final in that form. As the program developed the Harvard workers readily realized that the problem of setting up an absolute scale over a range of all attainable magnitudes presented more difficulties than caught the eye at the first glance.

First of all the stars chosen for the standards had to be of the same color in order to conform to the same scale. The Henry Draper Catalogue had been underway, and spectra of stars down to magnitude 8 could be accurately typed. Fainter stars were not left entirely to

*H.C.=Harvard Circular.

guess work. It was possible to characterize them as white, yellow or red down to a certain limit. Very faint ones had to be left almost entirely to conjecture. Even with the known colors of stars, Pickering recognized from long experience with photometric problems that the absorption characteristics of telescope lenses could introduce systematic errors which could destroy the objectivity of the standard and vitiate its use in reference to results obtained with other instruments. He therefore recommended that as many different kinds of methods and instruments as possible be used independently to check on one another and to give an accidental nature to otherwise systematic errors.

With the obvious need of great precaution in mind the final adopted North Polar Sequence consisted of 46 stars, all nearly of the same color, with an additional sequence of 12 reddish stars and a group of 38 supplementary stars. The 46 stars were numbered 1, 2, 3, etc.; the red stars, 1r, 2r, 3r, etc.; and the 38 supplementary ones, 1s, 2s, 3s, etc. In the main sequence all stars brighter than 11.4 are A type save one which is type F. Between $11^{\text{mag.}}$ and $16^{\text{mag.}}$ the stars are relatively white. For those fainter than $16^{\text{mag.}}$ the colors are not known. The stars of the red sequence vary from type G to type M. This great number of stars facilitated the derivation of the absolute scale and insured greater precision in its extension from star to star.

Thirteen telescopes, varying in aperture from the 0.5″ Ross-Zeiss to the 60″ Mount Wilson reflector were used for some 700 different exposures on 288 plates taken on the Pole, and on 10 plates taken on the Pleiades and Praesepe. These latter were made as a precautionary check lest the absolute value of the scale for the Pole be influenced by coloring of the stars.

The variety of instruments easily covered the entire range from the brightest to the faintest stars with plenty of overlap between the plates of the different telescopes. All exposures could be made sufficiently short to avoid atmospheric difficulties.

The plates were divided into groups, all of which were handled independently and weighted. This method of grouping provided a control over the vast quantity of material so that defective applications of methods and instrumental vagaries could be quickly recognized.

As Professor Pickering had foreseen, the most accurate results could only be assured by varying the methods used for the derivation of the absolute scale. Six different methods for reducing the intensity of the light from the stars by amounts which could be computed or accurately measured in the laboratory were adopted. The basic prin-

ciple behind all these was to cut down the light of a brighter star until its reduced image matched the unreduced image of a fainter star. Then in general,

$$m_f = m_b + C$$

where m_f signifies the apparent magnitude of the faint star, m_b the apparent magnitude of the brighter one and C the reduction constant corresponding to the method used. These constants were determined in terms of magnitudes according to Pogson's scale. Although almost any magnitude interval Δm could be found with the methods used, the interval best suited for keeping scale errors at a minimum was about 3 magnitudes.

The six methods adopted at Harvard were as follows:

1. *Extra-focal images.* The photometric laws governing the size and density of such images are well known. Brightness depends on the law that the intensity is inversely proportional to the distance from the focal plane. The light from different stars can be equalized by changing this distance and in the same way the brightness can be changed by known amounts. The ratio of the area of an image to the area of the free aperture of the objective provides an accurate determination of the reduction of light for images of different areas. This method was used by Professor E. S. King in setting up the scale for the bright end of the North Polar Sequence. (H.A. 76) Because the light of the stars is spread out so much in an extra focal image, this method cannot be used for stars fainter than 9 m.

2. *Iceland Spar plates.* The principle of this method has been used in the visual polarizing photometer. When the light of a star passes through a pair of doubly refracting plates, the intensity of the transmitted beam is a function of the angle between the axes of the plates. This dependence is explained by the fact that the intensity is proportional to the square of the amplitude of the light wave, and in double refraction the amplitude is reduced by increasing the angle between the axes. The difference between the light transmitted with the axes parallel and that transmitted with them at an angle ϕ is given by the relation

$$\Delta m = 2.5 \log \tan^2 \phi$$

In photographing stars through two such plates in the program of the North Polar Sequence, each star gave four images. Generally two of these images were badly distorted and could not be used.

3. *Absorbing screens.* A wire mesh screen placed over the objective or in front of the photographic plate, would reduce the intensity of point light sources by the amount

$$\Delta m = 10 \left[\log (a + b) - \log b\right]$$

where a signifies the thickness of the wires, and b the interval between the wires. In the application of this method at Mount Wilson, Seares preferred to determine the reduction constant for the screens experimentally by means of a constant light source and a Lummer-Brodhun photometer, (Mt. W.C.* #80) because other factors besides the dimensions of the mesh seemed to be involved.

Besides wire mesh screens other light absorbing media were used, for example, a perforated screen over the objective, shaded glass or a piece of photographic film over the plate. The constants for these all had to be determined experimentally in the laboratory.

4. *Half apertures.* One half of the aperture was covered so that theoretically in the same exposure time the light reaching the plate from a star would produce an image only one half as intense as the full aperture image.

5. *Circular diaphrams.* Placed over the objective these reduced the intensity in proportion to the reduced effective area of the lens.

$$\Delta m = 2.5 \log \frac{A_a}{A_o}$$

where A_a signifies the partial aperture area and A_o the full area.

6. *Auxiliary Prism.* A small prism with very slight dispersion was fastened to the center of the objective. The primary image of each star was then accompanied by a faint prismatic companion displaced to one side. The reduction constant could again be determined theoretically from the ratio of the area of the prism to that of the objective. This method, on account of the small area of the prism gave large reduction constants amounting to about 5 magnitudes. It had the advantage of requiring only a single exposure.

These methods of reducing the intensity of star images by known amounts were sufficiently independent of one another. But much care and caution had to be exercized in applying them. Sources of error in some of them were obvious, for example the variations in the absorption over the objective which changed the theoretical constants for the auxiliary prism and diaphrams. These errors had to be deter-

*Mt.W.C.=Mount Wilson Contributions.

mined as far as possible, but in many instances it was hoped that they would be small enough to be neglected and that they would eventually smooth out in the final means for all the methods.

Most of the methods implied the use of equal successive exposures on the same plate. An exposure without any absorbing screen or diaphram gave the so-called primary images, and the succeeeding exposure with the absorbing medium gave the secondary or reduced images. Since the stars were assumed to be of the same color, not much attention was given to the differential absorption for different colors. One bad feature in the use of multiple exposures was found. The sensitivity of a photographic plate changed very rapidly in the beginning of an exposure, so that another factor besides the reduction constant affected the results. This was a serious error and had to be eliminated as far as possible by reversing the order in which primary and secondary images were taken on successive plates.

The distance correction also proved troublesome and the best results were obtained only when the measures were confined to the immediate center of the plate. Seares determined the distance correction for the plates taken with the 60″ Mount Wilson reflector and these were used in the final reduction of the plates from this instrument at Harvard. His method consisted of a series of equal exposures of the same stars on different parts of the plate. The variation of the images of the same stars could then be measured as a function of the distance from the center of the plate. He found that with increasing distance the images of bright stars became larger and those of faint stars smaller.

The measurement of the plates was all done by means of a comparison scale, except in the case of extra-focal images. By comparing faint primary images with similar secondary images the difference in magnitudes between the bright and faint primary images became immediately known. The comparison scale is designed for comparing these images of the stars. It consists of a series of graded exposures on the same star, each successive image in the series differing by a factor of 2 or 3 in the exposure time. In general the factor 3 gave differences of about one magnitude between the images and the factor 2, about 0.8 mag. With a little experience the observer could estimate the size of a star image to within a tenth of the interval between two images of the scale. A scale was prepared for each telescope used so that the images on the scale would always appear comparable with those on the plates.

Seares has shown (Mt. W. C. #80) that the intervals of the comparison scale can be reduced to magnitudes by the simple relation,

$$b = \frac{\Delta m}{\Delta s}$$

where Δs signifies the difference in comparison scale corresponding to Δm computed for a set of primary and secondary images of a star. Once the factor b is determined,

$$m = a + bs - \Delta m$$

or

$$m = a + bs$$

depending on whether s signifies the measure of a secondary or a primary image. The constant a is determined by the zero point and Δm is the reduction constant.

The above method was not used at Harvard for deriving the absolute scale of magnitudes, although the graphic method used would yield a mean value for b. The method followed at Harvard was known as process A, and was an adaptation of the procedure for calibrating thermometers suggested by Professor S. W. Holman (A. J. S.* 123, 278).

From 84 plates, 20 series of magnitudes were derived at Harvard. An absolute scale was determined from each plate independently. The zero point which was not essential at that stage of the program in nearly every case was based on a provisional scale of photometric magnitudes. The average deviation in the absolute scale was $\pm 0.03^{mag.}$ and in the final mean, ten plates having average deviations exceeding $5 \times 0.03^{mag.}$ were rejected.

Once the absolute photographic scale had been established, a large number of measurements were made from other plates and their relative magnitudes based on the new absolute scale. These helped to reduce the accidental errors of the absolute scale. A further test was made by setting up absolute scales independently in the Pleides and in Praesepe to detect any influence of coloring in the scale derived from the polar stars. This check resulted in a correction of $+ 0.048^{mag.}$ to each of the final magnitudes listed in H.C. 170.

Since the absolute scale was determined with thirteen different telescopes, each of which contributed some portion to the entire range of the scale, the color equation of each had to be known and all reduced to some standard, before an arbitrary zero point could be adopted. All observations were reduced to the 1 inch Cooke and the 8 inch Draper telescopes at Harvard. This final scale along with the other incidental corrections constituted the homogeneous photographic scale.

The arbitrary zero point was defined by international agreement. It was so fixed that the photographic magnitudes would equal on the average the photometric magnitudes of the Harvard scale for stars having spectra of Class AO between $5.5_{mag.}$ and $6.5_{mag.}$

* A.J.S.=American Journal of Science.

Although Harvard devoted considerable effort and time to providing the world with a Standard Sequence, it was not alone in the task. Five other observatories made determinations of an absolute scale for the same sequence stars. Foremost among these was Mount Wilson where F. H. Seares with the 60″ reflector obtained a very accurate photographic scale and provided much helpful criticism in the revision of the final results at Harvard. He found for example that the Harvard absolute scale diverged from the Mount Wilson scale at the bright and faint ends of the sequence. The explanation of these divergences could not be given fully, although Seares was inclined to attribute the divergence at the faint end to a combination of space reddening and the method used in the final reduction at Harvard. (Mt. W.C. #80 & #81). In general the agreement of the scales was very good. Between $4^{mag.}$ and $7^{mag.}$ there was a divergence of about 2%, while between $7^{mag.}$ and $10^{mag.}$ the number of stars was too small to satisfactorily eliminate accidental differences. From $2.6^{mag.}$ to $10^{mag.}$ the mean error was only about $+ 0.03^{mag.}$ The stars fainter than $16^{mag.}$ had been observed only with the 60″ at Mount Wilson and it was desirable that these be observed elsewhere with a different instrument to provide a valid comparison. His final conclusion was that for stars brighter than $16_{mag.}$ there will be little need for adjustment of the absolute scale in the future.

Besides deriving an independent photographic scale, Seares also contributed an excellent photovisual scale for the North Polar Sequence which agrees remarkably well with the Harvard visual scale. Both scales coincide at $6^{mag.}$ and at $12^{mag.}$ Obviously color causes some difficulty in the intervening intervals. It extends to $17^{mag.}$ (A table of these magnitudes is given in the H. der Ap.* Band II/2, pages 495 et s.) The photovisual and photographic scales for Mount Wilson are mutually consistent at far as $17^{mag.}$ the limit of the photovisual scale. The maximum difference is $0.09^{mag.}$ at $10^{mag.}$ The photographic scale extends to $20^{mag.}$

By analysing the results from five different observatories, Seares obtained a weighted mean deviation for each. The following table contains these deviations along with the title of the program, its author and the references in which the work is discussed.

The following abbreviations have been used.

Gr. signifies Greenwich
HI signifies Harvard magnitudes by Miss Leavitt, 10^m—16^m
HII signifies Harvard magnitudes by Professor King, brighter than 9^m.

*H der Ap.=Handbuch der Astrophysik.

Program	M. Dev.	Author	Reference
Gottingen	$0^m.038$	Schwarzschild	Gott. Actinometrie
Gr. I	0.028	Chapman & Melotte	M.N.74,1913; Mt.W.C.#25.
Gr. II	0.025	S. Jones	M.N. 82,1921
HI	0.019	Leavitt	H.A. 71
HII	0.029	King	H.A. 76
M.W.60″	0.022	Seares	Mt.W.C.#97 & #235
M.W.10″	0.025	Seares & Humason	Mt.W.C.#234 & #235
Potsdam	0.041	Schwarzschild & Dziewulski	A.N. 198, 1914
Yerkes	0.047	Parkhurst	Ap.J. 36, 1912

It is clear from the above table that the work done by Miss Leavitt at Harvard was especially precise. Independent comparisons were made at Harvard with the results from Potsdam, Mount Wilson, Greenwish, Gottingen and Yerkes. These warranted certain corrections to the final results. The zero point was corrected by $0.08^{mag.}$ The resulting magnitudes were published in H.A. 71.*

Further critical studies of the results and data showed a few slight irregularities still extant in the Harvard scale. Most of the discussions centered on the reduction to a standard color system. The final corrections were published in H.B.* 781. The average correction from $10^{mag.}$ to $15.5^{mag.}$ was about $0.25^{mag.}$ Stars fainter than $16^m.2$ are rejected as standards until further checked by other independent observations. The early part of the scale has a slight correction advancing gradually from $-0.14^{mag.}$ at $2.5^{mag.}$ to $0.00^{mag.}$ at $7^{mag.}$ and thence to $0.18^{mag.}$ at $9.5^{mag.}$ A table of these corrections is given in H.B. 781 and in the mimeographed notes by Mrs. Gaposchkin on the North Polar Sequence.

The final scale may be considered accurate for all purposes. The zero point adopted in 1922 with reference to A0 stars was found to be vitiated by about $+ 0.1^{mag.}$ due to space reddening by a nebula at the pole. Hence the zero point was not strictly applicable to A0 stars in other parts of the sky. The previous corrections given in H.B. 781 necessitated a correction to all Harvard Magnitudes published prior to 1922. Further corrections would have made matters too complicated. It was therefore proposed in the 1938 meeting of the I.A.U.* that the zero point defined in 1922 be more or less abandoned for photographic magnitudes and that the North Polar Sequence as it now stands be accepted as defining the zero point for the International Photographic Scale. This simply means that the color index of A0 stars will no longer be $0.00^{mag.}$ but $-0.14^{mag.}$

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*H.A.=Harvard Annals.

*H.B.=Harvard Bulletin.

*I.A.U.=International Astronomical Union.

It is evident that not all of the difficulties had been anticipated in selecting the north polar region for a standard sequence. The primary aim was to establish a set of standard magnitudes which could be reached at all times of the year. It was also intended to extend this standard to all of the Harvard Standard Regions and the Kapteyn Selected Areas. Several disadvantages have been recognized, among which the most troublesome have been the presence of absorption in the north polar cap and the fact that photographic images of polar stars are noticeably different from those of equatorial or low latitude stars. This latter fact makes it difficult to transfer the sequence in polar comparison plates.

Nevertheless the precision of the results is a lasting memorial to the many scientific workers who devoted so many years to the trying routine of measuring and reducing plates, and to extracting every tangible error from a vast mass of observational material.

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MATHEMATICS

THE FOUR FUNDAMENTAL OPERATIONS OF ALGEBRA

Rev. Conrad Bilgery, S.J.

Current text books of College Algebra advance a variety of reasons for the validity of these four fundamental operations.

Some authors base their proof on concrete applications of algebraic numbers, such as loss and gain, opposite directions of positive and negative numbers, and so forth.

But Algebra is primarily an abstract science, and the laws of the four fundamental operations must logically flow from the abstract nature of algebraic numbers.

Kantian mathematicians realize this fully, but they have no concept or definition of an abstract negative number. Hence, in Kantian fashion they have recourse to arbitrary assumptions.

Let the reader open the College Algebra by Jos. B. Rosenbach and E. A. Whitman (Ginn & Co. 1933). On page 5 he meets the significant title, "Fundamental Assumption of Algebra."

Now, as to the explanation:

> "The four fundamental operations of Algebra are addition, subtraction, multiplication and division, and these operations are extensions to a larger class of numbers of the same operations in arithmetic. In order that these operations may be extended to general numbers, certain fundamental laws governing the use of general numbers are necessary. These are in the nature of assumptions, since it is not possible to prove them. They are known as the fundamental assumption of Algebra, and play much the same role in Algebra as the postulates do in Geometry."

The conclusion is evident:

"Algebraic truth is based on assumption."

Of course, these Kantian modernistic mathematicians admit that said laws are convenient and in this sense very plausible.

In his address, "Infinity and Non-Euclidean Geometry," the writer showed at the Chicago convention in 1940 the nugacity of these Kantian modernistic geometers.

Positive numbers of Algebra follow as such the laws of Arithmetic, but before we can establish the laws of negative numbers in combination with positive numbers, we must first determine the manner in which abstract negative numbers are derived from the numbers of Arithmetic.

The answer is simple: We arrive at a negative number if we attempt to substract a greater number from a smaller number.

e. g.:

$$5 - 4 = 1$$
$$5 - 5 = 0$$
$$5 - 6 = 5 - 5 - 1 = 0 - 1 = -1$$
$$5 - 7 = 5 - 5 - 2 = 0 - 2 = -2$$
$$a - (a + x) = a - a - x = 0 - x = -x$$

From these negative remainders we come to the evident definition of an abstract negative number. A negative number is a loose combination of two ideas. Thus (-5) expresses the absolute value of the arithmetical 5, and we attach to it the operation of subtraction, i.e., 5 is to be subtracted from 0, or from any other value if the latter be on hand.

In the abstract, the minus sign of a number does not express a quality, but is the symbol of the operation of subtraction.

For the sake of emphasis, especially in concrete applications of algebraic numbers, the pure number of arithmetic is adorned with the plus sign and is called a positive number.

From this follows the new or algebraic definition of zero, for zero is now the sum of two numbers, opposite in sign but of the same absolute value. Thus, $(+a) + (-a) = 0$

In reality this means that $a - a = 0$

In the abstract a positive number is simply a number of Arithmetic and the plus sign can be omitted, while a negative number is a number of Arithmetic to be subtracted.

1. Algebraic addition.

   $+5 + (+7) = 5 + 7 = 12$
   $(-5) + (-7)$ means that 5 and 7 are both to be subtracted, or, all in all, 12 is to be subtracted.
   Hence, $(-5) + (-7) = -12$
   Hence, Rule 1. Numbers of like signs are added by taking the sum of their absolute values and prefix the common sign to the result.

Again:

$$(+12)+(-7)=5+7-7=5+0=5$$
$$(-12)+(+7)=(-5)+(-7)+(+7)=-5+7-7=-5$$

Rule 2. Numbers of unlike signs are added by finding the difference of their absolute values and prefixing to the result the sign of the greater absolute value.

2. Algebraic subtraction.

$(+12) - (+7) = 12 - 7 = 5$
or $[5 + (+7)] - (+7) = 5$
or $12 + (-7) = 12 - 7 = 5$
$(-12) - (-7) = [(-5) + (-7)] - (-7)$, i.e.,
take away $(-7$ from $[(-5) + (-7)] = -5$
Also: $(-12) + (+7) = -12 + 7 = -5$
Rule: Change the sign of the subtrahend and add algebraically.

3. Algebraic Multiplication.

a) If the multiplier is positive:
$(+12) . (+3) = 12 . 3 = 36$
$(-12) . (+3) =$ Three times $(-12) = -36$
b) If the multiplier is negative:
$(+12) . (-3)$. Here the multiplier implies the idea of multiplying by 3 substractively.
$\therefore$ substract $(+12) . 3$ from $0 = 0 - 36 = -36$.
$(-12). (-3)$. Take (-12) three times and substract, i.e., subtract $(-12) . 3$ from 0.
$\therefore (-12) (-3) = 0 - (-36)$
$0 = 36 + (-36)$ from which we must take away or remove $(-36) = +36$.
Rule: In algebraic multiplication like signs give + and unlike signs give — .

4. Algebraic Division.

a) $$\frac{(+12)}{(+3)} = \frac{12}{3} = 4$$

$$\frac{-12}{+3} = \frac{1}{3}(-12) = -4$$

b) If the divisor is negative:
In that case, the divisor again implies a double process,— that of division and that of subtracting the result.

$$\frac{+12}{-3} \text{ i.e., take } \frac{1}{3} \text{ of } 12 \text{ and subtract it from } 0.$$

$$\therefore \frac{+12}{-3} = -\left\{\frac{1}{3} \text{ of } 12\right\} = 0 - 4 = -4.$$

$$\frac{-12}{-3} \text{ i.e., subtract } \frac{1}{3} \text{ of } (-12) \text{ from } 0 = 0 - (-4)$$

$0 = (+4) + (-4)$, and taking away (-4) there is left $+ 4$.

Rule: In algebraic division like signs give $+$ and unlike signs give $-$, as in case of multiplication.

CONCLUSION:

The laws of the four fundamental operations of Algebra are logically derived from the abstract notion of negative numbers and the nature of the numbers of Arithmetic. Hence, these laws are not based on assumptions, but on self-evident truths.

N. B. Shortly after the submission of this article to the Jesuit Educational Quarterly, Father Bilgery died at Denver, August 12, 1945. R. I. P.

PHYSICS

THE CLASSICAL THEORY OF DISPERSION

Joseph F. Mulligan, S.J.

Two previous articles have treated of the exponential solutions of free, damped and forced electrical oscillations.* In this article the method previously outlined will be applied to the dispersion of light.

The electromagnetic theory of light explains dispersion, (i.e., the variation of index of refraction with frequency), on the basis of the electrical properties of matter. Matter is composed of atoms made up of a positively charged nucleus and extra-nuclear electrons. The nucleus is a compact group of protons and neutrons. The protons are positively charged particles, while the neutrons are electrically neutral. The number of protons gives the atomic number of an element, while the combined number of protons and neutrons gives the mass number of the element. The charge on the electron is equal and opposite to the charge on the proton, and since the atom is electrically neutral, the number of extranuclear electrons is also equal to the atomic number. These extra-nuclear electrons are capable of vibrating with definite natural frequencies about the nucleus. Some of these natural frequencies might be expected to be of the order of magnitude of frequencies encountered in ordinary light. If this were so, we would expect that these vibrating electrons would influence the passage of light through matter, and would result in the phenomenon called dispersion. That this model gives us a first approximation to the true picture is indicated by the close agreement of experimental observations with theoretical predictions.** A more exact theory of dispersion demands the use of the quantum theory.

We will confine this discussion to dispersion in gases, for here the interaction between atoms (or molecules) is small enough to be neglected. Our procedure will be first to consider the equation of motion of the electrons under the influence of the light wave, then apply Maxwell's equations to obtain the equation for the propagation of the light wave through the gas, and finally investigate the relation of the index of refraction to the frequency of the light wave.

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* Robert O. Brennan, S.J., "The Exponential Solutions of the Equations of the Oscillator," *A. A. J. S. Bulletin,* September and December, 1945.

** Cf. Wood, *Physical Optics,* pp. 490 ff.

In the case of a dielectric, in this case a gas, in which there are no free charges and hence no current density due to the movement of free charges, Maxwell's first equation for the electromagnetic field takes the form, for the x-component:*

$$Dy(W) - Dz(V) = (K/c)\ X \qquad (1)$$

where K is the dielectric constant of the gas, c is the velocity of light in a vacuum, X is the x-component of the electric intensity vector E, and V and W are the y and z-components of the magnetic intensity vector H.

This equation expresses the time rate of change of the x-component of the electric field in terms of the space rates of change of the y and z-components of the magnetic field. As yet, however, this equation is not complete. Although there are no free charges present in the gas, and hence no flow of free charges, the passage of the light wave through the gas causes something similar to a current flow. The light wave displaces the negatively charged electrons with respect to the centers of positive charge. This so-called polarization of the medium results in a displacement of bound charges which is equivalent to a current flow. Suppose there are N electrons per unit volume in the gas, each with the same mass m and the same charge e. If $\dot{x}$ is the average velocity of these electrons in the x direction due to the polarization of the medium, then $N\dot{x}$ pass through a unit area at right angles to the x direction per second, and the displacement of charge due to this movement is $Ne\dot{x}$. This is equivalent to the current density u. Since according to Maxwell's theory a current in the gas would introduce a term $4\pi u/c$ on the right hand side of equation (1), to account for the polarization of the medium, equation (1) must now take the form:

$$Dy(W) - Dz(V) = (4\pi Ne/c)\dot{x} + (K/c)\dot{X} \qquad (2)$$

Equation of Motion of Electrons

We suppose that the electrons are subject to an elastic restoring force proportional to the displacement x, and to a damping force proportional to the velocity. A light wave of angular frequency w (2π times ordinary frequency) passing through the gas subjects each electron to a sinusoidal external electric field, the x-component of whose intensity is:

* Throughout this paper we will use Dy(W) as meaning the partial derivative of W with respect to y. Second partial derivatives, for example with respect to x and y, will be written $D^2xy(W)$. First and second partial derivatives with respect to the time will be written in the familiar form $\dot{x}$ and $\ddot{x}$.

$$X = Xo \exp(jwt) \qquad (3)$$

Since the charge on the electron is e, the force acting on each electron is eX. The resulting motion for each electron is then, for the x-coordinate:

$$m\ddot{x} + mR\dot{x} + \underline{w}^2 mx = eXo \exp(jwt) \qquad (4)$$

where $\underline{w}$ is the natural angular frequency of the electron and mR is the damping factor.

Since the right hand side of (4) is complex, it is clear that x must also be complex. Throughout our entire discussion we will continue to work with complex quantities of the type $X = Xo \exp(jwt)$. When we have obtained a solution in complex form, the real part of that solution will be the part which has physical significance, and in which we are interested.

It is to be expected that the frequency of the resultant motion of the electron will be the same as the frequency of the impressed light wave, and so we take as a trial solution $x = A \exp(jwt)$, where A is complex, and proceed as in the solution of the forced oscillator. If we neglect the "transient" part of the solution which dies down too quickly to be of importance, our solution takes the form:

$$x = \frac{eXo \exp(jwt)}{m(\underline{w}^2 - w^2 + Rjw)} \qquad (5)$$

This equation indicates the "steady-state" motion of the electron. We are more interested however in discovering how this motion of the electron reacts on the light wave and affects its propagation through the gas.

Propagation of Light Wave

Replacing $Xo \exp(jwt)$ by its value from equation (3), we can rewrite equation (5) as:

$$m(\underline{w}^2 - w^2 + Rjw)x = eX \qquad (6)$$

where both sides are still in complex form. We now want to combine this with equation (2). To do this we multiply through by Ne and obtain:

$$m(\underline{w}^2 - w^2 + Rjw)Nex = Ne^2X$$

Differentiating with respect to the time and bringing the bracket on the left to the denominator on the right, we have:

$$N e \dot{x} = \frac{Ne^2/m}{w^2 - w^2 + Rjw} \dot{X} \qquad (7)$$

Substituting this value for $Ne\dot{x}$ in equation (2):

$$Dy(W) - Dz(V) = (1/c)\left(K + \frac{4\pi Ne^2/m}{w^2 - w^2 + Rjw}\right)\dot{X} \qquad (8)$$

Since the term in the second parenthesis is complex, we will for convenience let it equal $(n - jk)^2$. The physical significance of this term will be seen later. We then obtain:

$$Dy(W) - Dz(V) = [(n - jk)^2/c]\,\dot{X} \qquad (9)$$

By combining this equation with Maxwell's equations for the magnetic intensity, the following equation can then be derived:

$$\text{Lap } X - [(n - jk)^2/c^2]\ddot{X} = 0 \qquad (10)$$

where Lap X (=Laplacian of X) = $(D^2xx + D^2yy + D^2zz)\,X$

This equation can be shown to be the differential equation of a harmonic wave. The familiar equation for a transverse harmonic wave moving along a string placed on the x-axis is:

$$S = So \cos w(t - x/v) \qquad (11)$$

where S is the displacement, So the amplitude or maximum displacement, and v the phase-velocity of the wave. By successive differentiations we can derive from this equation the differential equation of the wave, which proves to be:

$$D^2xx(S) - (1/v^2)\ddot{S} = 0 \qquad (12)$$

Similarly the equation of a harmonic wave travelling in any arbitrary direction through space is given by the expression:

$$S = So \cos w[t - (fx + gy + hz)/v] \qquad (13)$$

where f, g, h are the direction cosines of the normal to the wave front. The differential equation of such a wave is:

$$\text{Lap } S - (1/v^2)\,\ddot{S} = 0 \qquad (14)$$

This equation is identical in form with equation (10), provided $c^2/(n - jk)^2 = v^2$. Since (14) is derived from a wave motion, it is obvious that equation (10) also represents a wave motion. Since (13) is a solution of (14), as can be proved by direct substitution, it is clear that a solution of equation (10) takes the form:

$$X = X_o \cos w[t - (n - jk)\ (fx + gy + hz)/c] \quad (15)$$

where, it will be remembered, X is the x-component of E, the intensity of the electrical field due to the light wave. We can write (15) as an exponential if we realize that we are considering only its real part:

$$\begin{aligned} X &= X_o \exp jw[t - (n - jk)\ (fx + gy + hz)/c] \\ &= X_o \exp[-wk(fx + gy + hz)/c] \\ &\qquad \exp jw[t - n(fx + gy + hz)/c] \end{aligned}$$

This is our solution in complex form. To get a solution that represents the actual propagation of the wave through the gas, we take the real part of this, and obtain:

$$\begin{aligned} X &= X_o \exp[-wk(fx + gy + hz)/c] \cos \\ &\qquad w[t - n(fx + gy + hz)/c] \end{aligned} \quad (16)$$

By a comparison with (11) and (13) it can be seen that the cosine term represents the motion of a harmonic wave propagated through space. Since v in (11) and (13) represents the phase-velocity of the wave, it is clear that c/n has also the dimensions of velocity. Therefore $v = c/n$, or $n = c/v$. But this ratio of the velocity of light in a vacuum to its velocity in a medium is defined from Huygen's wave-front construction as the index of refraction of the medium. Hence n in (16) is the index of refraction of the gas through which the light wave is propagated.

The exponential term in equation (16) is a decay factor that decreases as $(fx + gy + hz)$ increases, that is, as the light wave is propagated through space. Hence as the light wave is propagated through the gas, its energy is absorbed and its amplitude decreases. The speed with which the amplitude decreases can be seen to depend on the factor k which is called the absorption coefficient.

The term $(n - jk)$ is often called the complex refractive index or the "dynamic" refractive index, but this terminology is somewhat misleading. This complex refractive index is rather a "doublet" composed of the real refractive index and of the absorption coefficient. When this "dynamic" refractive index of the medium is known, the optical properties of the medium are completely determined.

Relation of Index of Refraction to Frequency

To discover how the frequency of the light affects the absorption and refraction, we must consider the substitution previously made:

$$(n - jk)^2 = \left(K + \frac{4\pi N\ e^2/m}{\underline{w}^2 - w^2 + Rjw}\right)$$

Since for gases both K and n are very nearly unity, we can replace K by 1 and extract the square root of both sides. To do this we expand the right hand side by the binomial theorem, neglecting all terms after the second. This gives to a high degree of approximation:

$$n - jk = 1 + \frac{2\pi N\ e^2/m}{\underline{w}^2 - w^2 + Rjw}$$

Separating into real and imaginary parts by multiplying by the conjugate complex of the denominator, we obtain:

$$n - 1 = \frac{2\pi N(\underline{w}^2 - w^2)e^2/m}{(\underline{w}^2 - w^2)^2 + R^2w^2} \qquad (17a)$$

$$k - \frac{2\pi NRwe^2/m}{(\underline{w}^3 - w^2)^2 + R^2w^2} \qquad (17b)$$

These values for the index of refraction and absorption coefficient indicate how the frequency of the incident light affects the propagation of the wave through the gas. It can be seen that if the frequency of the light is very small or very large compared with the natural frequency of the electrons, k approaches zero, and hence the exponential term in equation (16) approaches a limiting value of unity, and the amplitude varies little from Xo. If however w is of almost the same magnitude as $\underline{w}$, k becomes large, attaining a maximum value of $2\pi Ne^2/mRw$ when $w = \underline{w}$. The two are then said to be in resonance. The die-away factor is then of great importance, and as the wave is propagated, its energy and amplitude rapidly decrease.

The value of n in equation (17a) indicates that for values of w which are very large or very small compared to $\underline{w}$, the index of refraction increases as w increases. However in the neighborhood of the

absorption region where w is of almost the same magnitude as $\underline{w}$, the factor R^2w^2 in the denominator becomes of importance and the index of refraction decreases rapidly with increasing frequency. When $w = \underline{w}$, $n = 1$. Since the velocity of the wave is, from equation (16) c/n, it is clear that the velocity of light varies not only with the medium but also with the frequency of the light. This variation of refractive index and hence also of velocity with frequency is what is meant by dispersion.

The values of k and n in equation (17) indicate a close connection between absorption and dispersion. If the natural frequency of the electrons lies in the infra-red or ultra-violet then there is little or no absorption over the range of the visible spectrum, and the index of refraction increases with increasing frequency. This is so-called normal dispersion. If however the electrons have a frequency which falls within the visible spectrum, then when light of nearly the same frequency as the natural frequency of the electrons passes through the medium, absorption takes place, and in the neighborhood of the absorption band the index of refraction decreases with increasing frequency. This anomalous behaviour gives rise to the term anomalous dispersion.

It is clear that the expressions for n and k in equation (17) hold only for the greatly simplified case in which the electrons in the gas are all characterized by the same frequency and the same damping factor mR. If there is more than one type of electron present, the index of refraction and absorption coefficient are given by a summation of terms similar to those in equation (17).

In conclusion it may be well to indicate the relationship between the index of refraction and absorption coefficient and the solutions of the forced oscillator. Since we took as our trial solution for equation (4):

$$\begin{aligned} x &= A \exp(jwt) \\ &= (a - jb)\ (\cos wt + j \sin wt) \end{aligned}$$

the real part of x is given by:

$$x = a \cos wt + b \sin wt \qquad (18)$$

where

$$a = \frac{Xo(\underline{w}^2 - w^2)e/m}{(\underline{w}^2 - w^2)^2 + R^2w^2} \qquad (19a)$$

$$b = \frac{XoRwe/m}{(\underline{w}^2 - w^2)^2 + R^2w^2} \qquad (19b)$$

If we compare the values of a and b with the values of n-1 and k in equation (17), we find that:

$$n - 1 = \frac{2\pi Ne}{X_o} a \qquad (20a)$$

$$k = \frac{2\pi Ne}{X_o} b \qquad (20b)$$

Thus the index of refraction and absorption coefficient are essentially related to the real and imaginary parts of the amplitude of the solution of equation (4), the equation that gives the motion of the electron under the impressed force of the light wave. From equation (18) it can be seen that a is the amplitude of the component of the motion of the electron in phase with the impressed light wave, while b is the amplitude of the component out of phase. The component in phase with the light wave results in an index of refraction different from unity. The component out of phase causes absorption and the diminution of the energy and amplitude of the light wave. If we were to plot the values of a and b against the angular frequency w of the light wave, we would obtain curves which are similar to curves a and b in figure I of the article on forced oscillations (*A. A. J. S. Bulletin*, December, 1945). Graphs of n-1 and k against the frequency of the light wave take essentially the same form.

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